

Fraktale Geometrie



Fraktale Geometrie

Gerade, Strecke: Dimension 1

Ebene, Fläche: Dimension 2

Küstenlinie von Großbritannien:
Dimension ?



Fraktale Geometrie

- Begriff: fractus (lat.) = gebrochen
 - bzw. Verb: frangere = zerbrechen
- „gebrochene“ Dimension



Fraktale Geometrie

Fraktal =

- Figur, in der jeder Teil dem Ganzen selbstähnlich ist
- hat unendliche Länge (bzw. Fläche)
- hat eine „gebrochene“ Dimension



Erzeugung eines Fraktals

Koch-Kurve

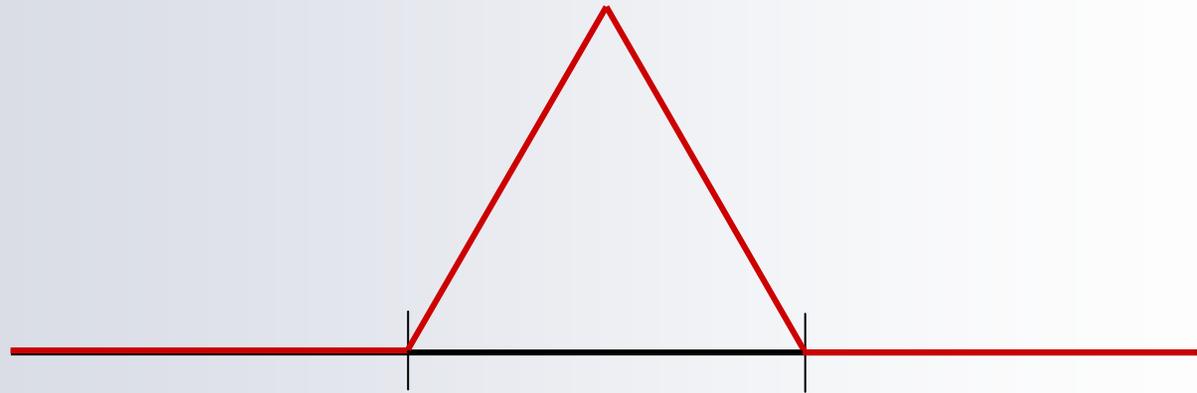


Initiator



Erzeugung eines Fraktals

Koch-Kurve



Initiator

Generator



Erzeugung eines Fraktals

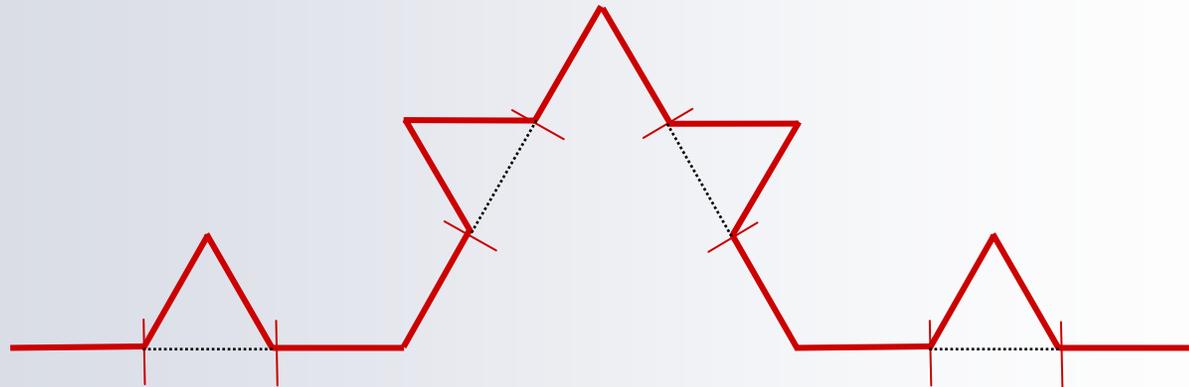
Koch-Kurve

1.Generation



Erzeugung eines Fraktals

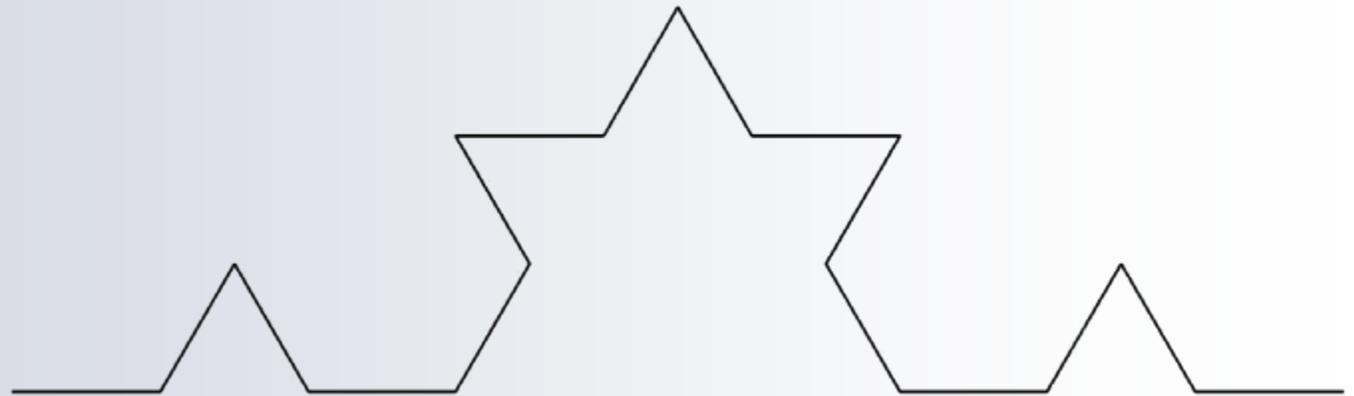
Koch-Kurve



Erzeugung eines Fraktals

Koch-Kurve

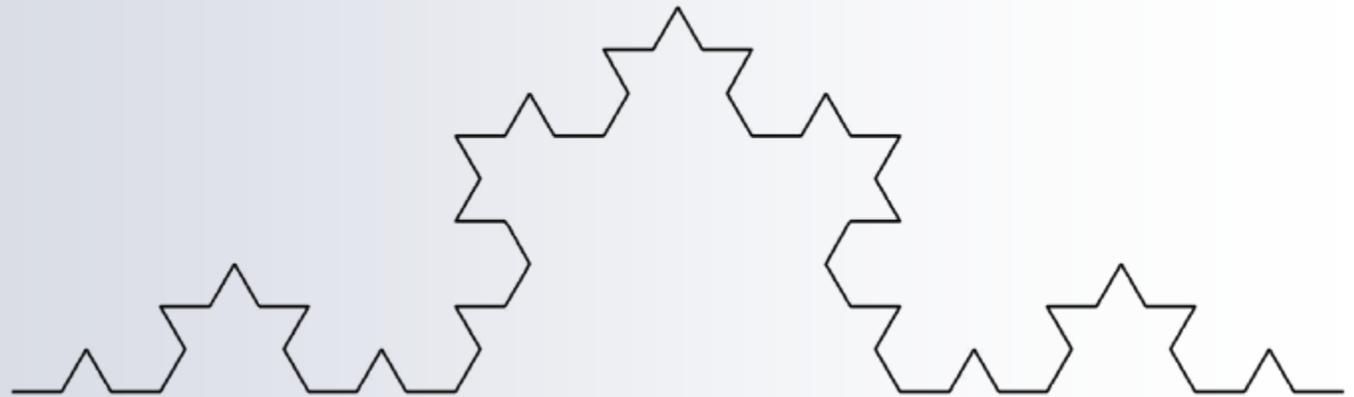
2.Generation



Erzeugung eines Fraktals

Koch-Kurve

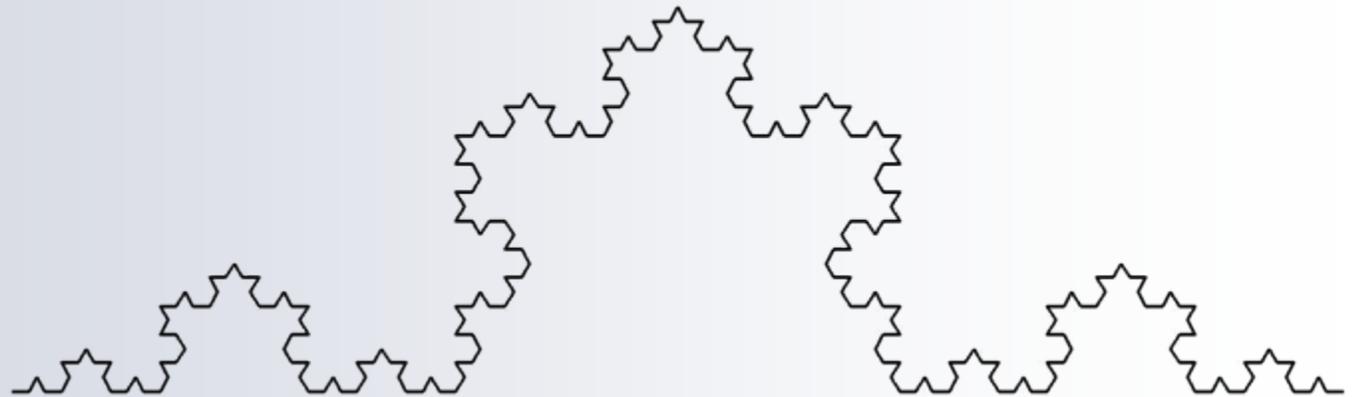
3.Generation



Erzeugung eines Fraktals

Koch-Kurve

4.Generation



Berechnung der Dimension

Initiator wird in s Teile geteilt
und durch N neue Teile ersetzt

Dann gilt:

$$N = s^D \quad \text{und damit} \quad D = \frac{\log N}{\log s}$$

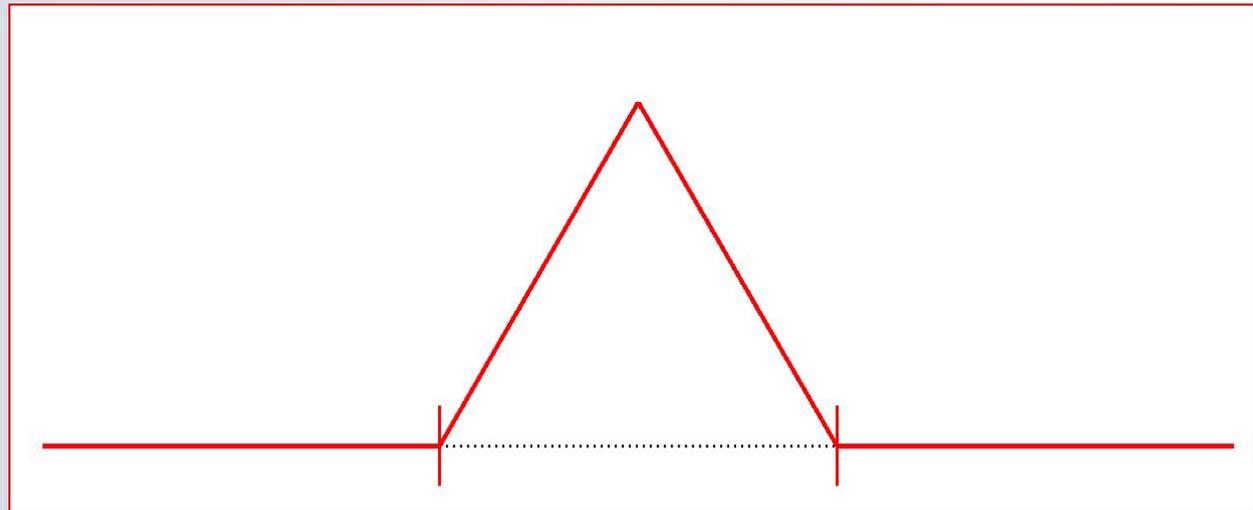


Berechnung der Dimension

Koch-Kurve

Linie wird in 3 Teile geteilt $\rightarrow s = 3$

Anzahl der Teilelemente $\rightarrow N = 4$



Berechnung der Dimension

Koch-Kurve

Linie wird in 3 Teile geteilt → $s = 3$

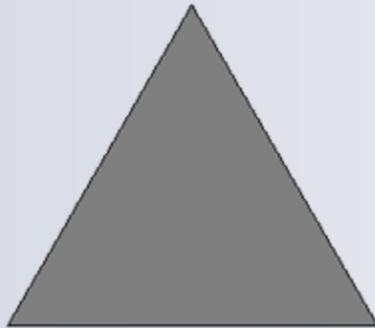
Anzahl der Teilelemente → $N = 4$

$$D = \frac{\log N}{\log s} = \frac{\log 4}{\log 3} = 1,2619\dots$$

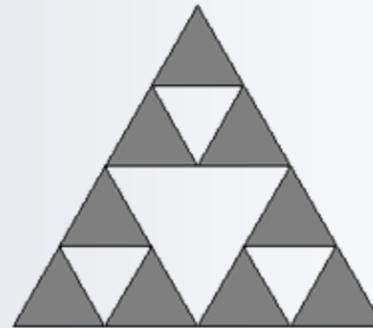


Beispiele für Fraktale

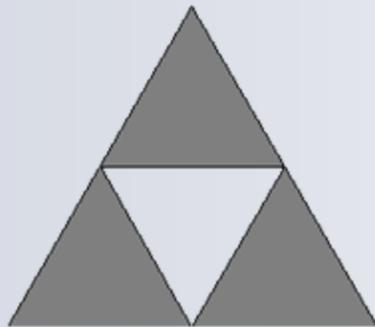
0 (Initiator)



2



1 (Generator)



6



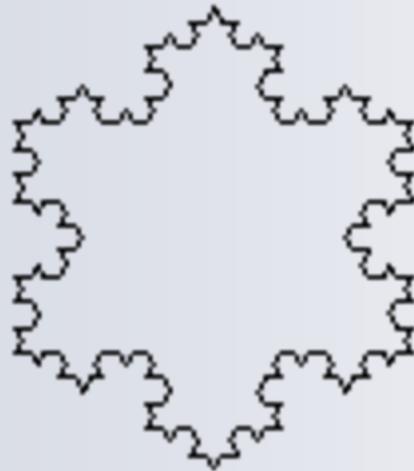
Sierpinski-Dreiecke

$D = 1,585$

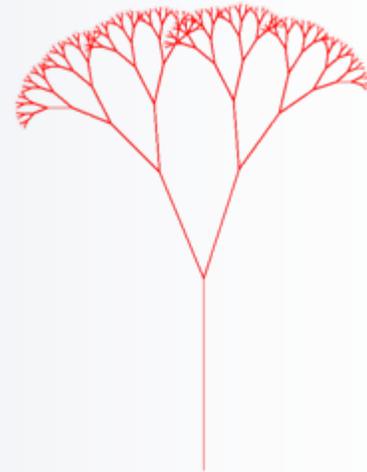


Beispiele für Fraktale

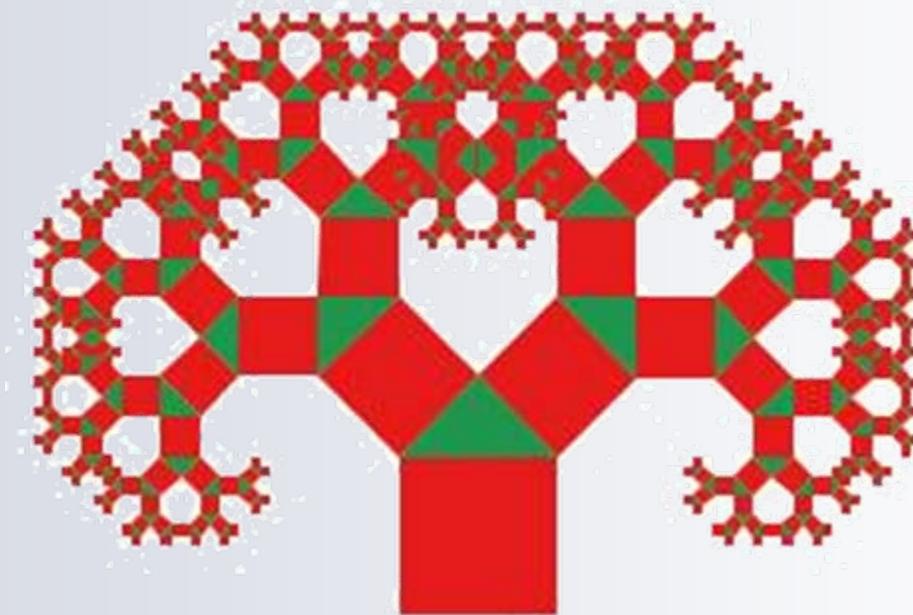
Schneeflockenkurve



Bäume



Beispiele für Fraktale



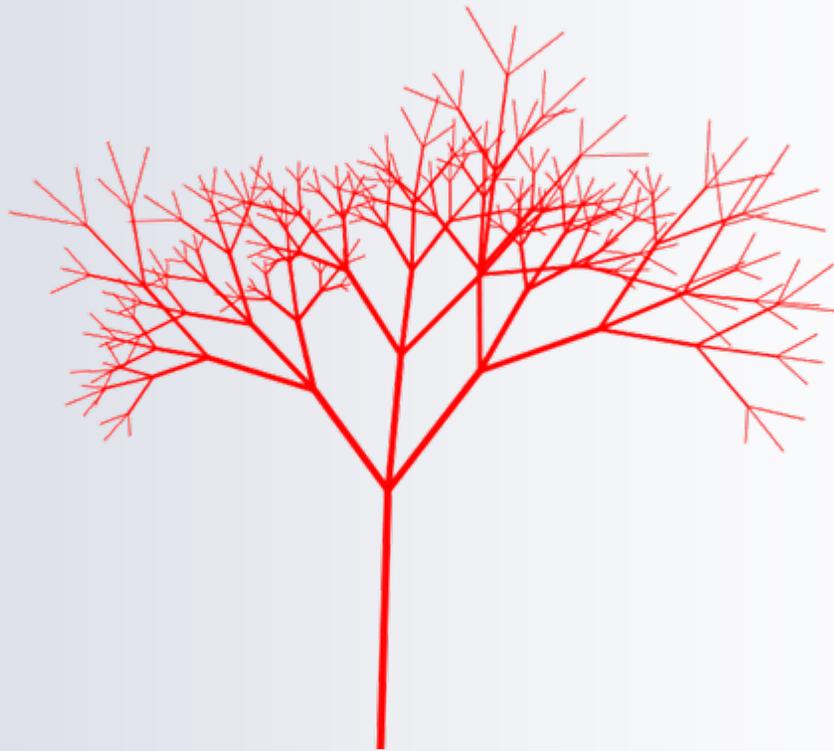
Pythagoras-Bäumchen



Beispiele für Fraktale



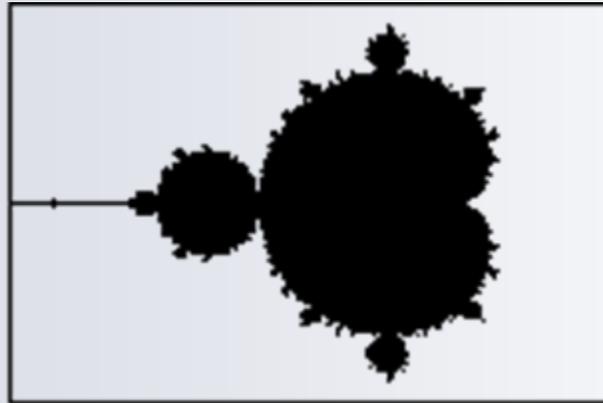
Beispiele für Fraktale



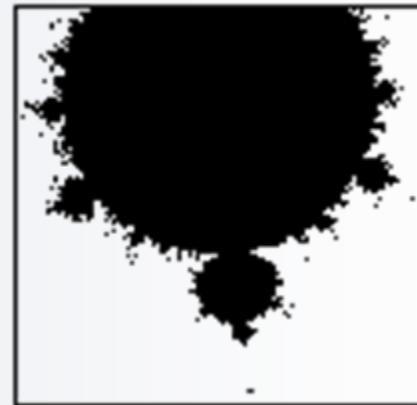
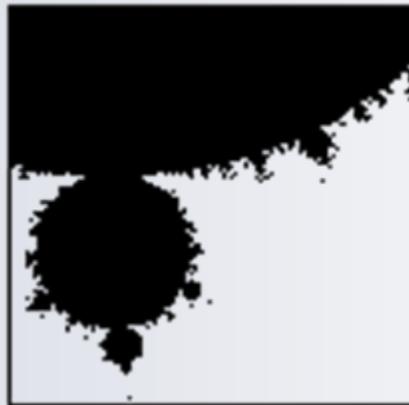
Beispiele für Fraktale



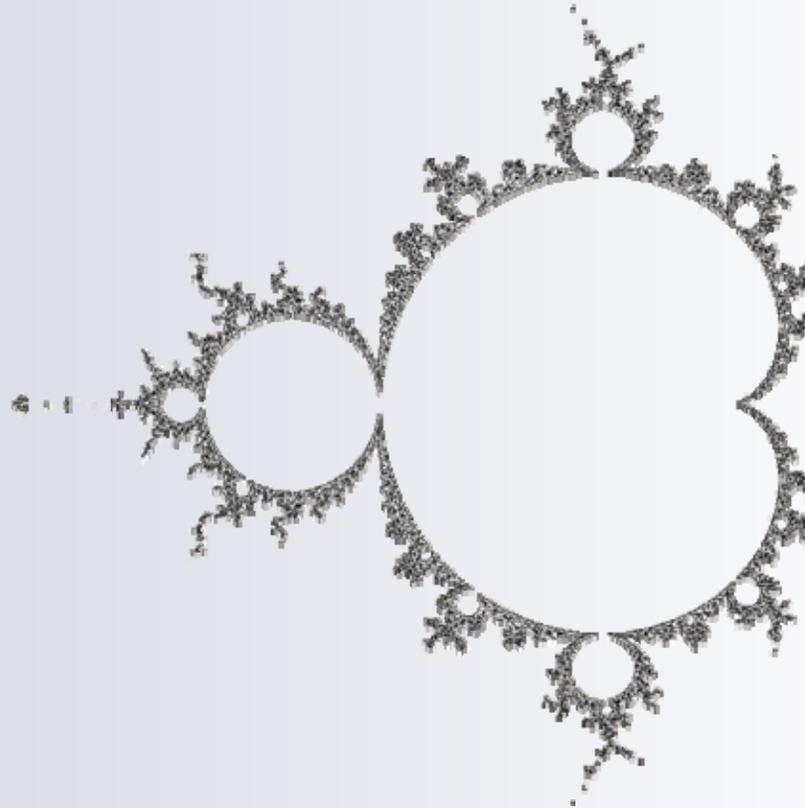
Beispiele für Fraktale



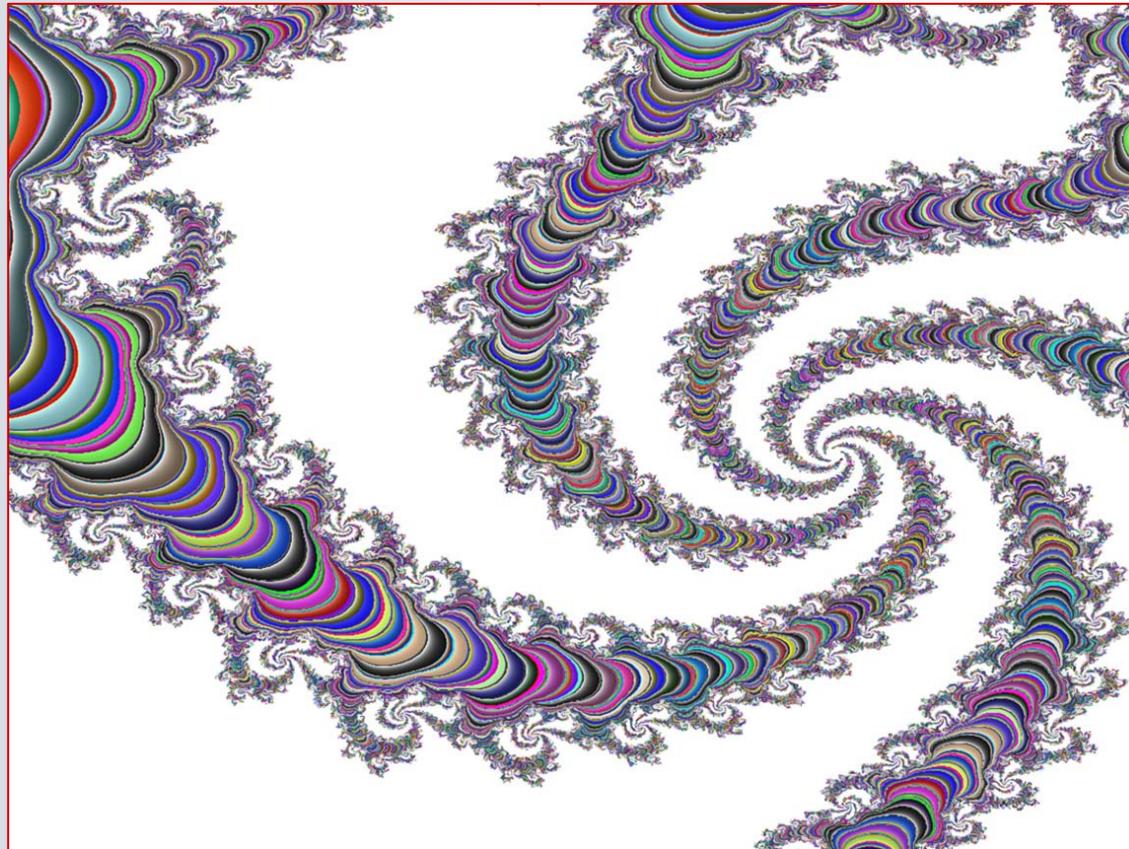
Mandelbrot-Menge
bzw. Apfelmännchen



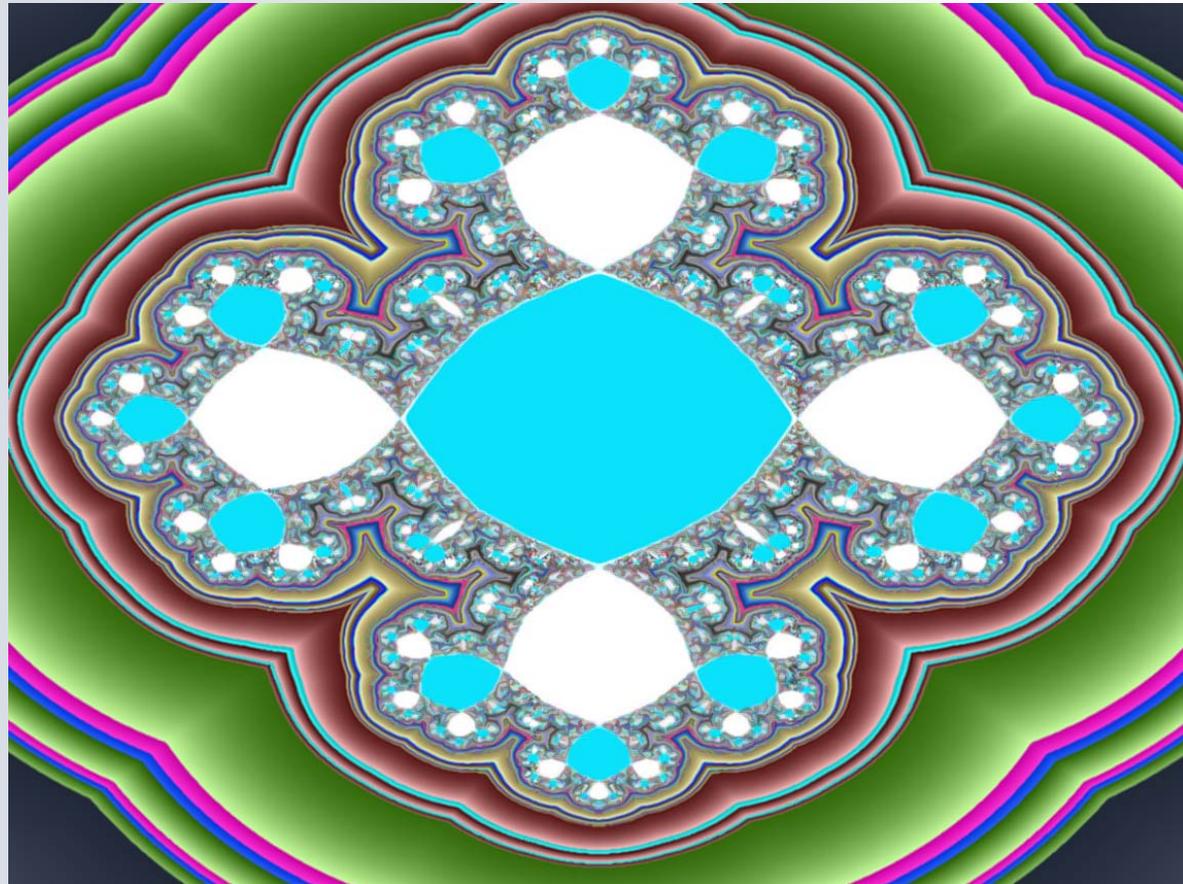
Beispiele für Fraktale



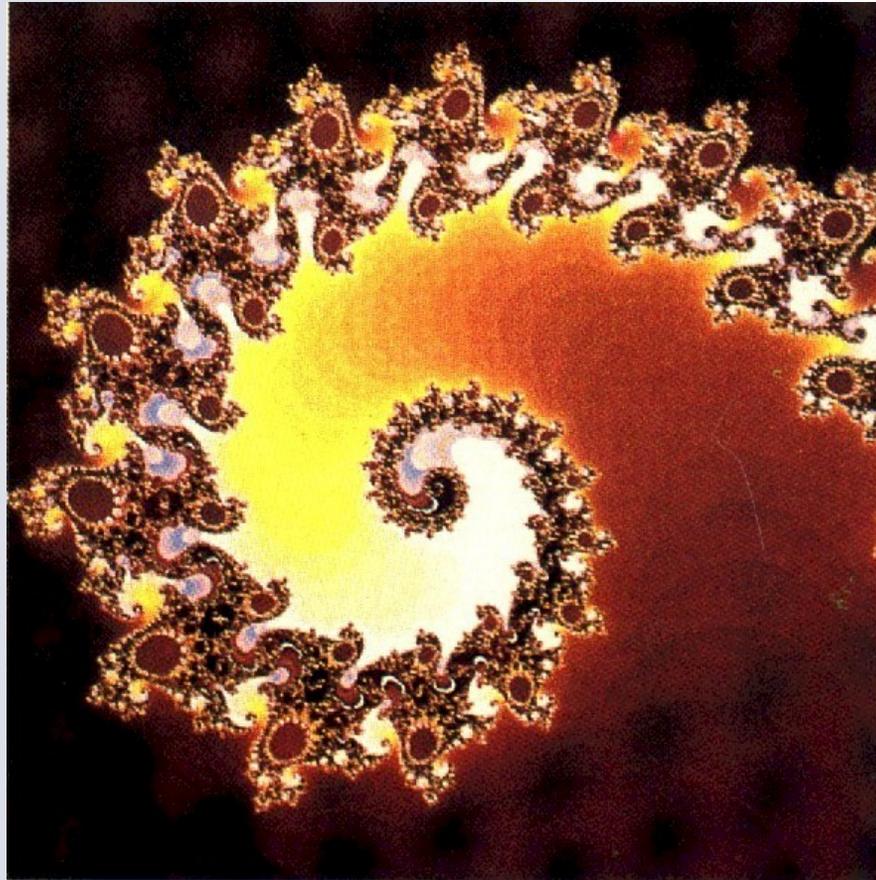
weitere Beispiele



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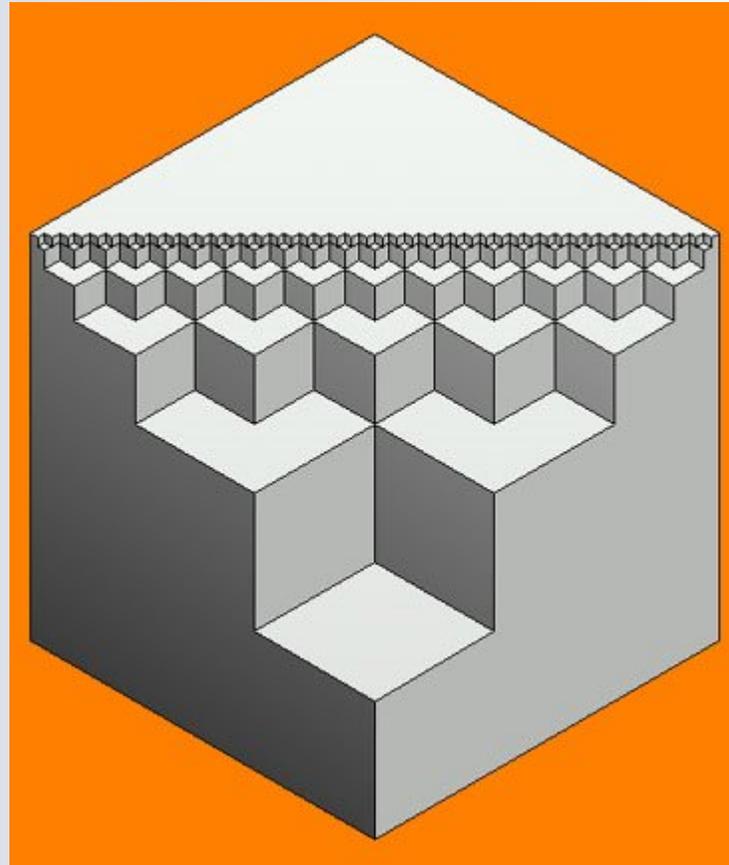
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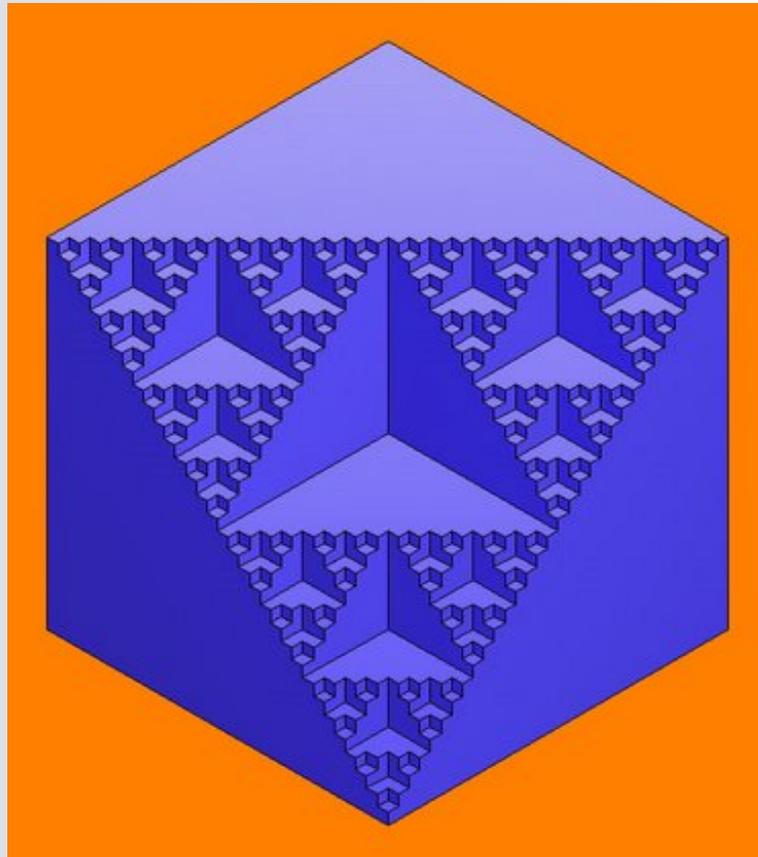
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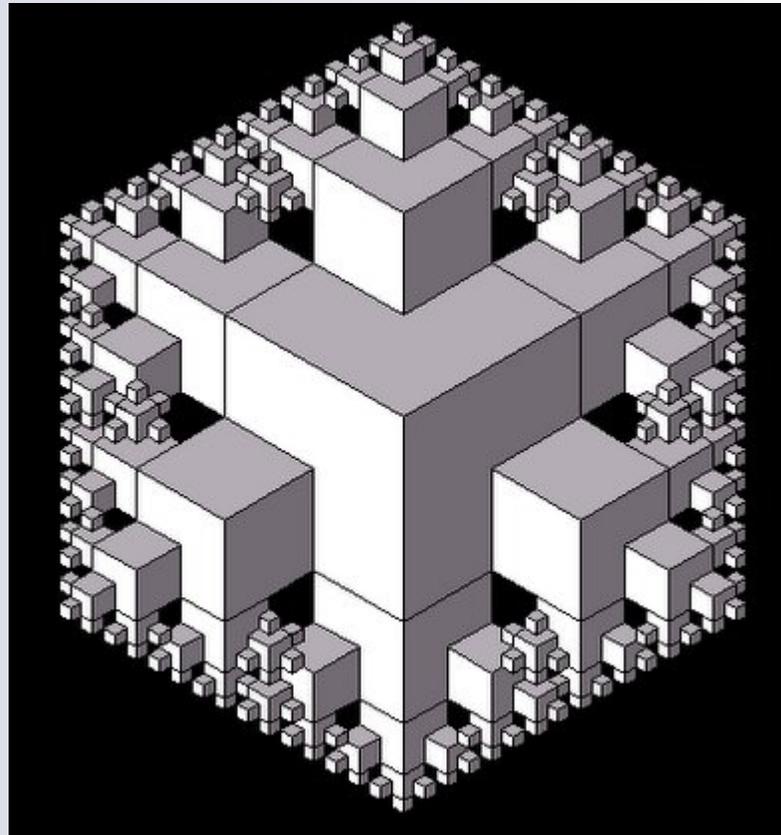
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Fraktale Geometrie

- Wichtige Vertreter:
 - Benoît Mandelbrot
 - Waclaw Sierpinski
 - Gaston Julia

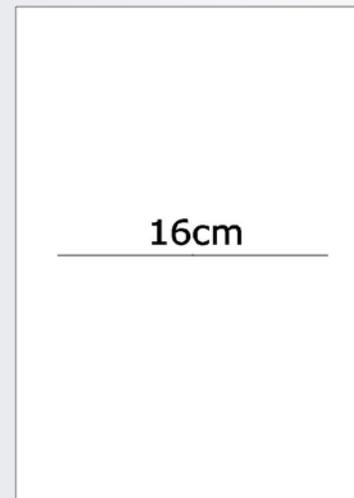


Pop-up Cards



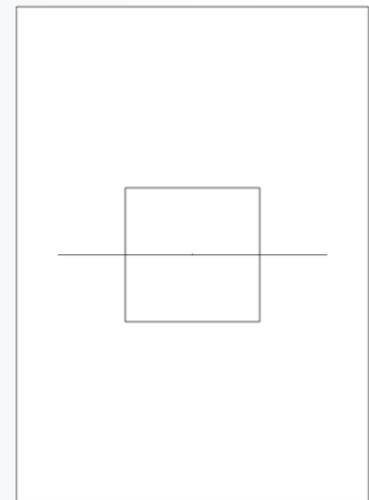
Pop-up-Card

- Nehmen Sie ein DIN A4-Blatt zur Hand
- Halten Sie es im Hochformat und
- zeichnen Sie in der Mitte eine horizontale Linie von 16 cm
- → **INITIATOR**



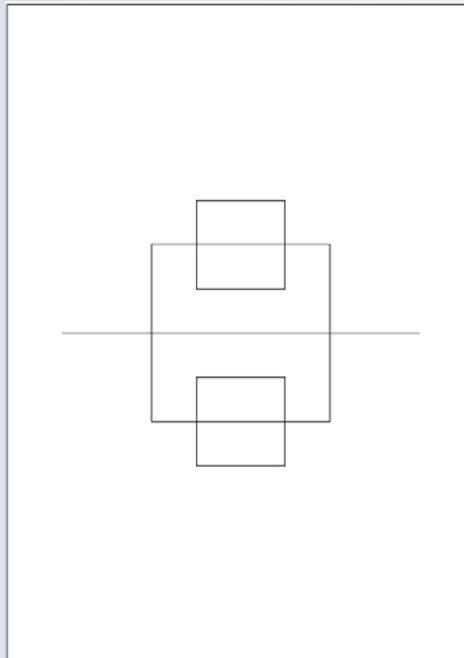
Pop-up-Card

- Teilen Sie diese Linie im Verhältnis 1:2:1, dh 4cm – 8cm – 4cm
- Ersetzen Sie die mittlere Teilstrecke durch ein Quadrat, dessen Mittellinie die mittlere Teilstrecke ist
- → **GENERATOR**



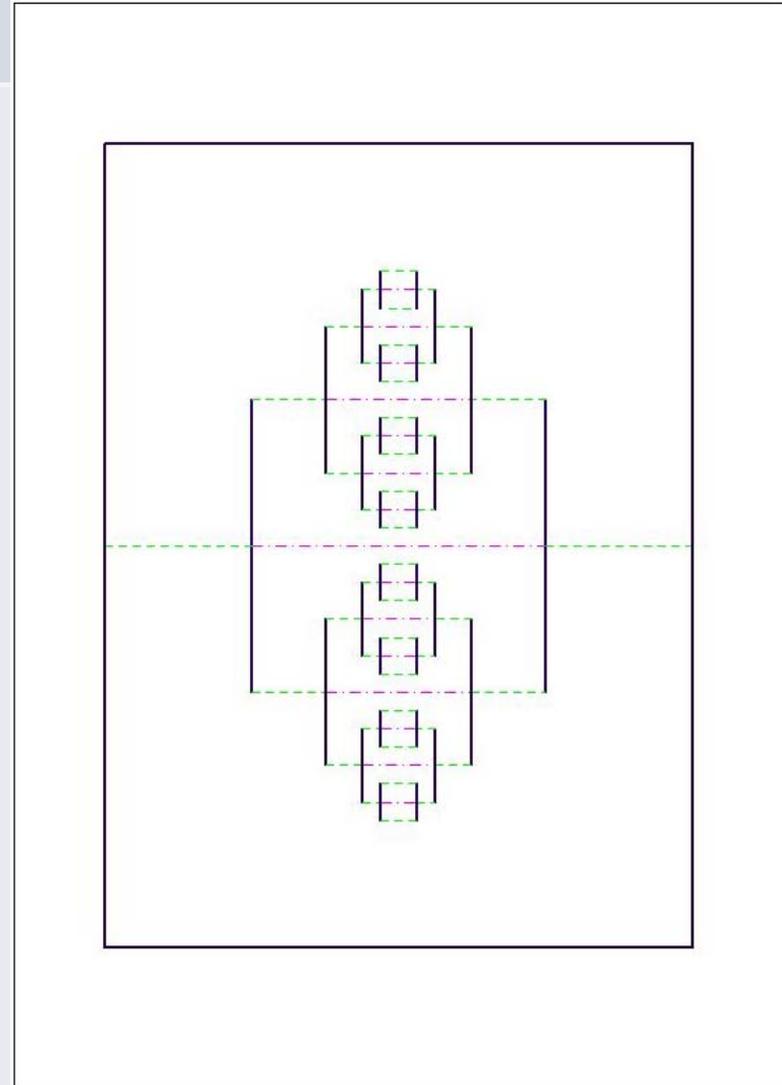
Pop-up-Card

- Die beiden waagrechten Quadratseiten werden wieder geteilt...



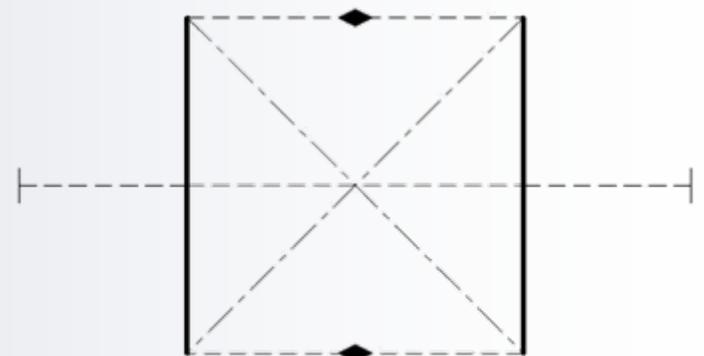
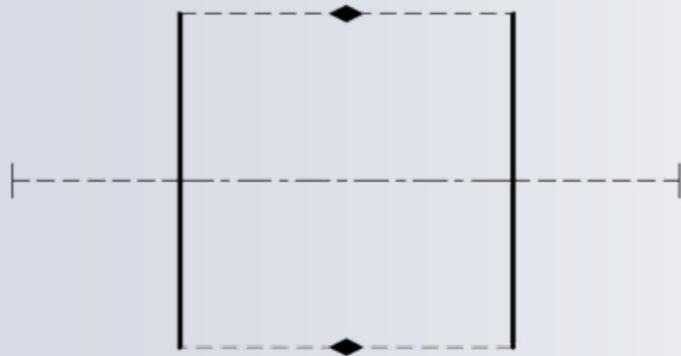
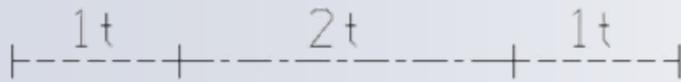
Pop-up-Card

- eventuell die Linien gestalten

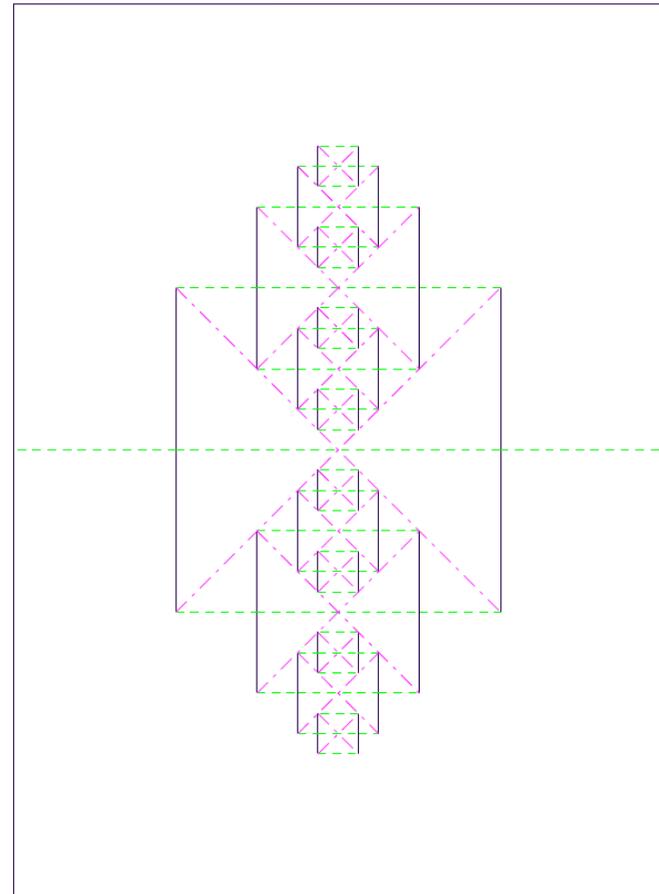
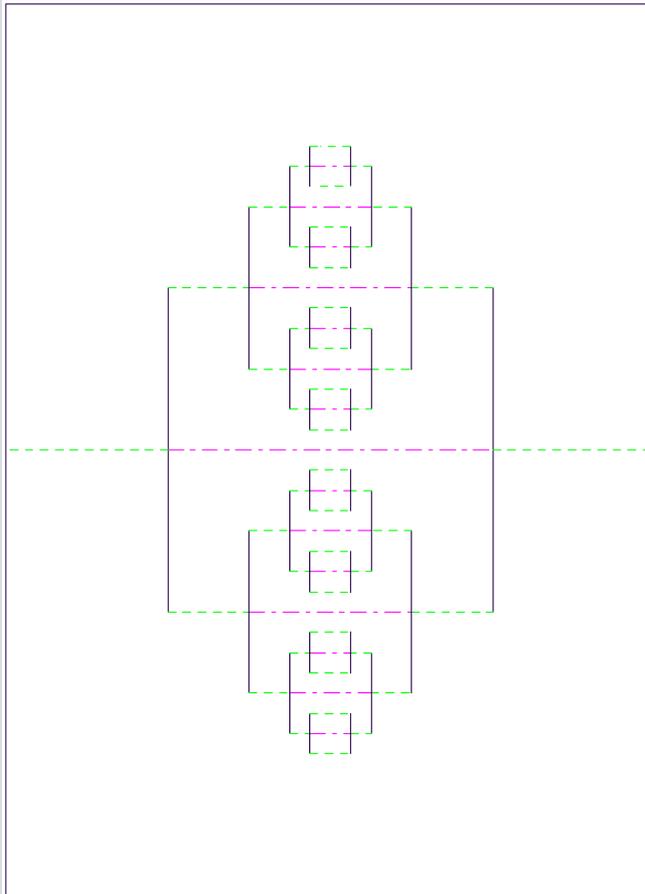


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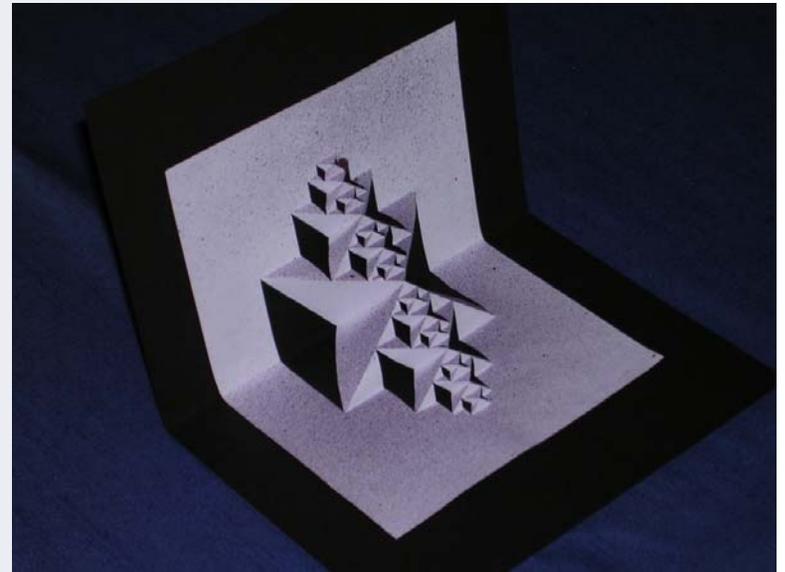
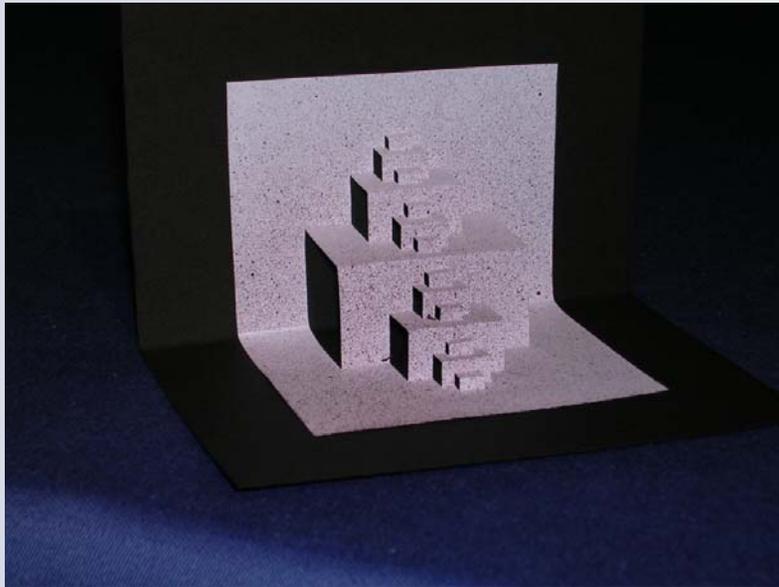
■ Varianten



Popup-Card

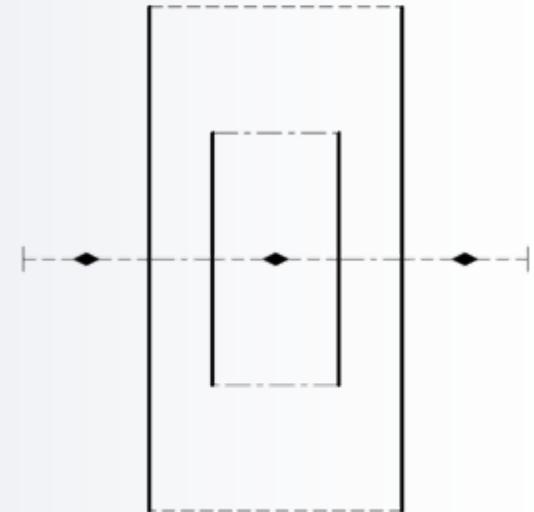
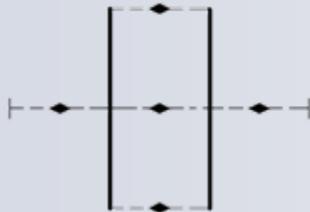
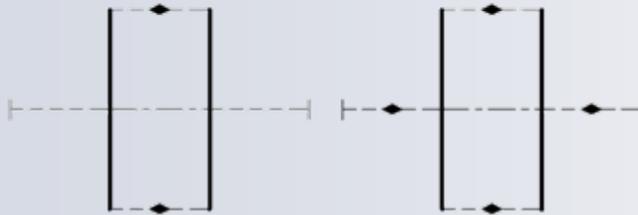


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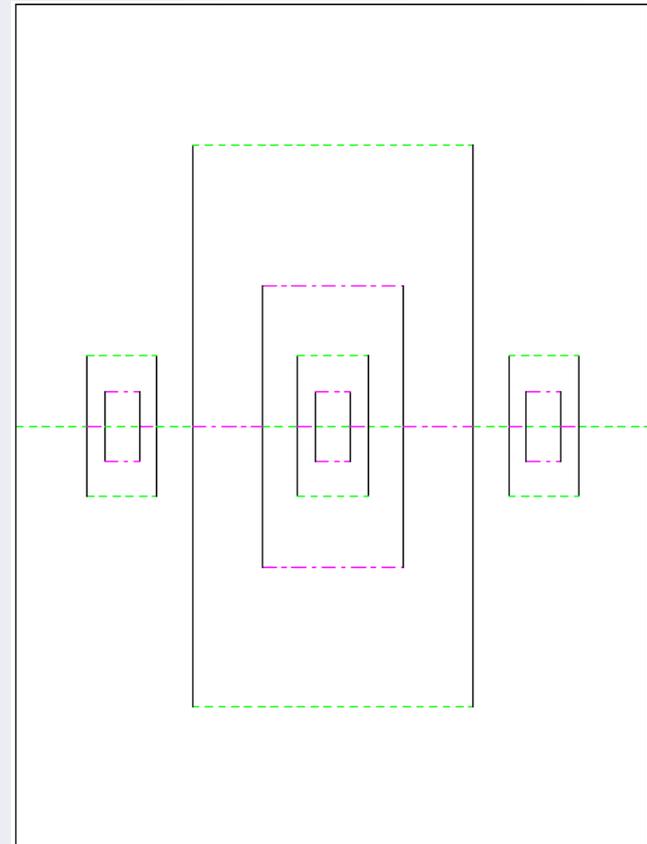
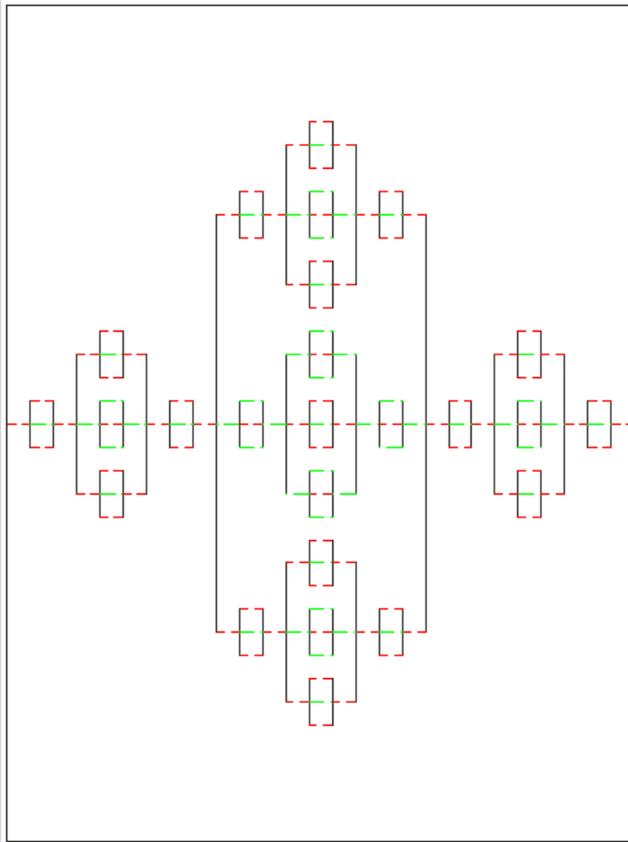


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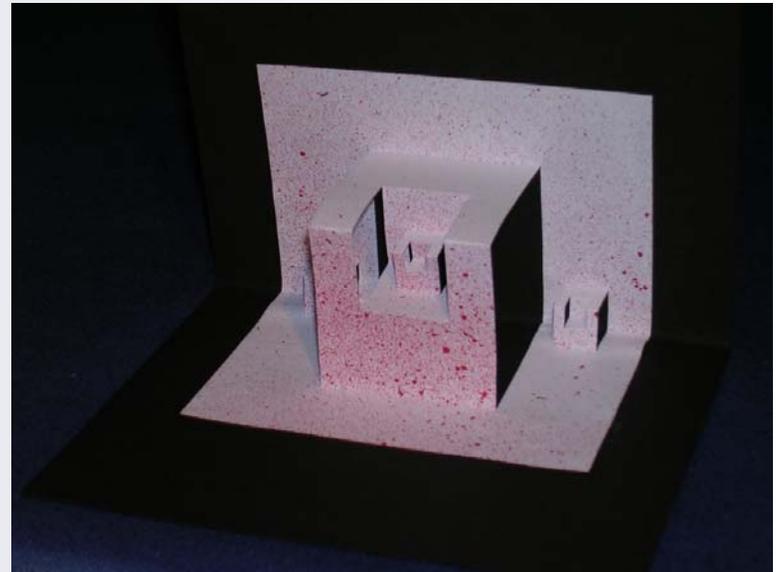
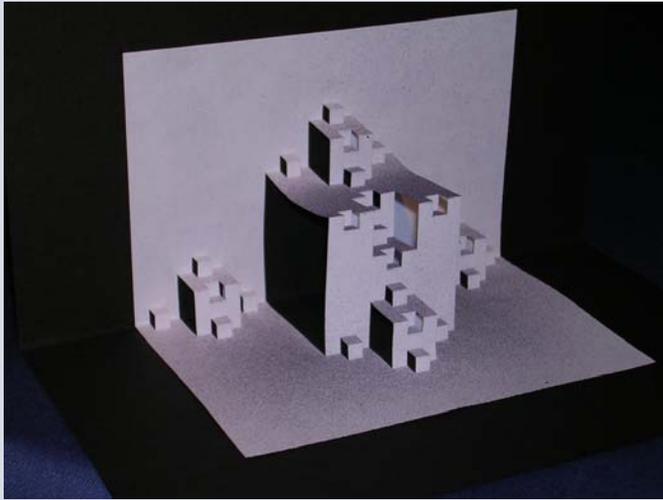
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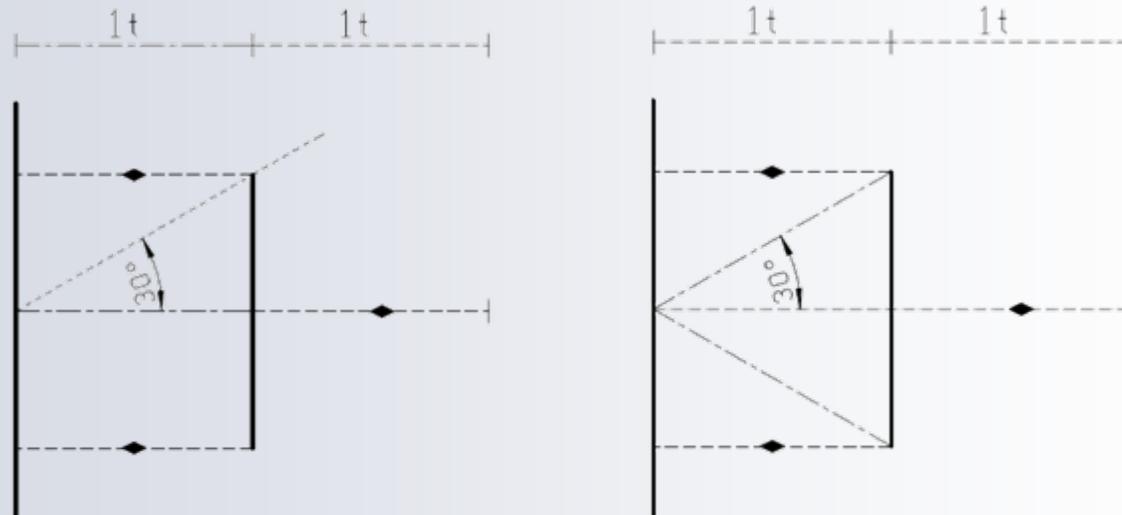


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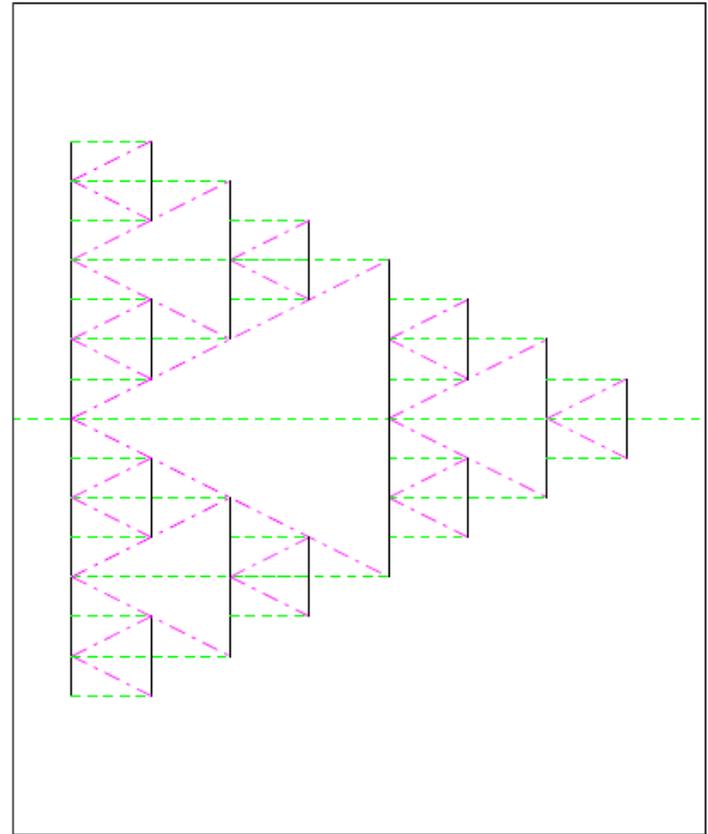
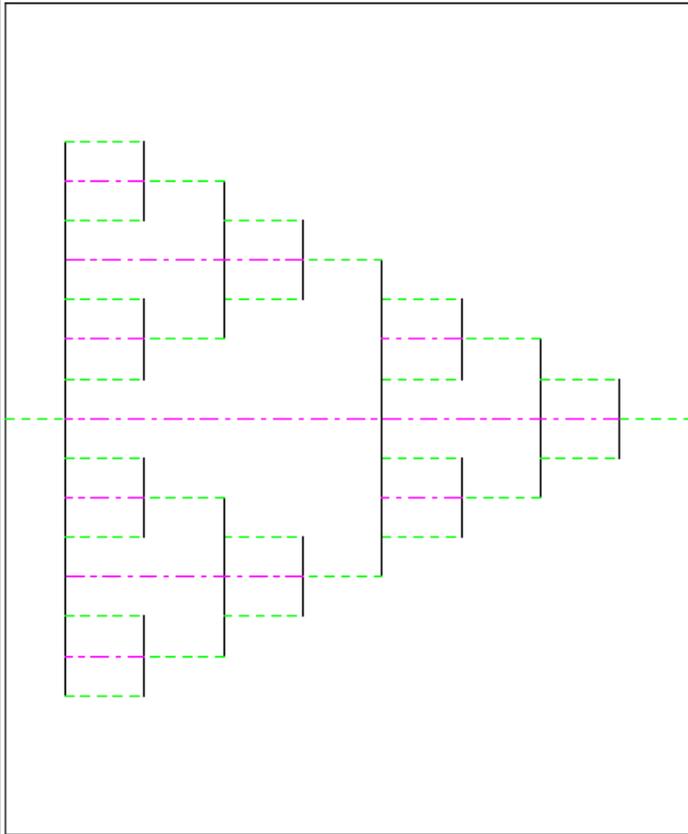


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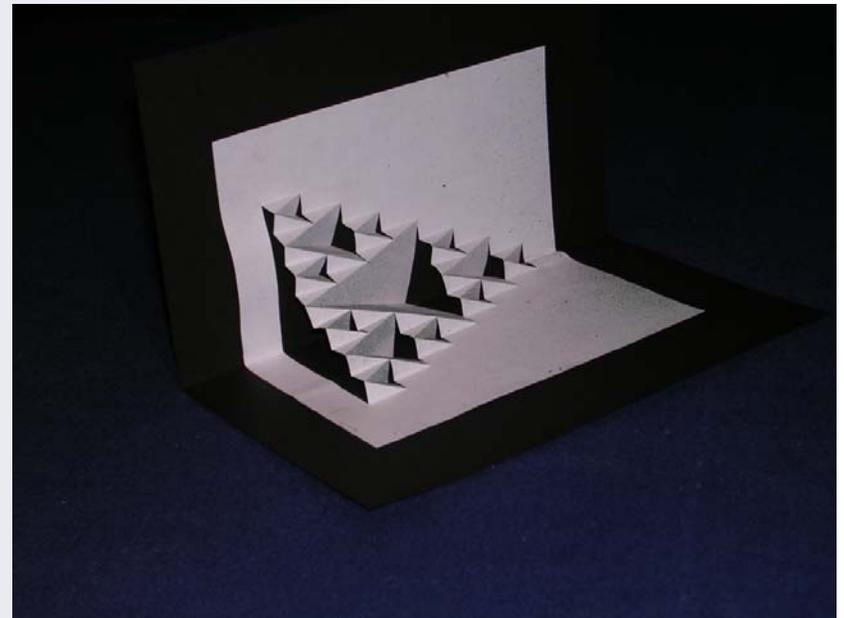
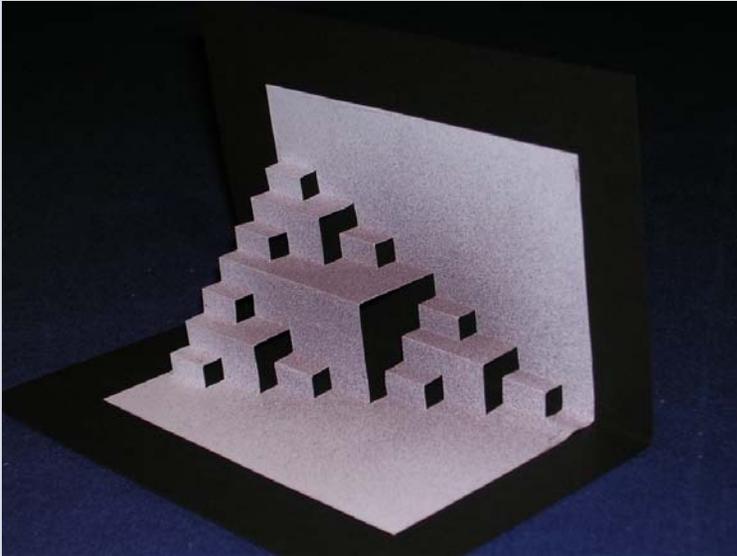
■ Varianten



Pop-up-Card



Popup-Card

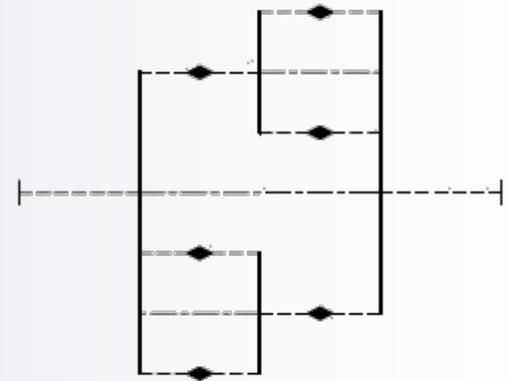
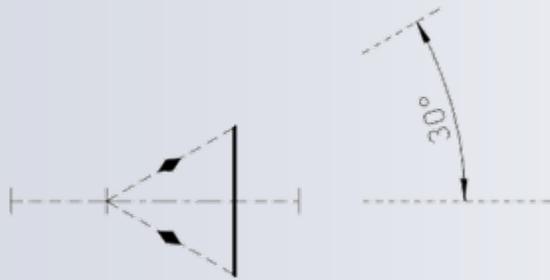


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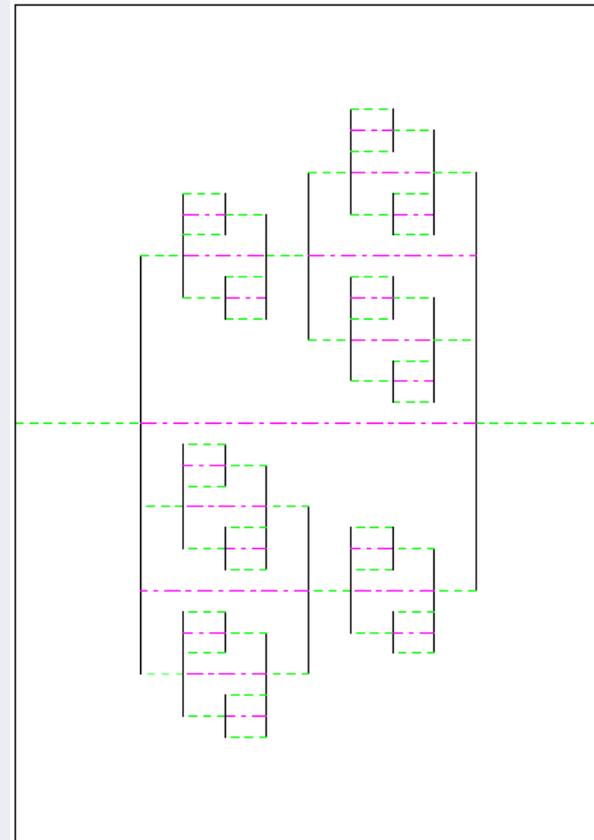
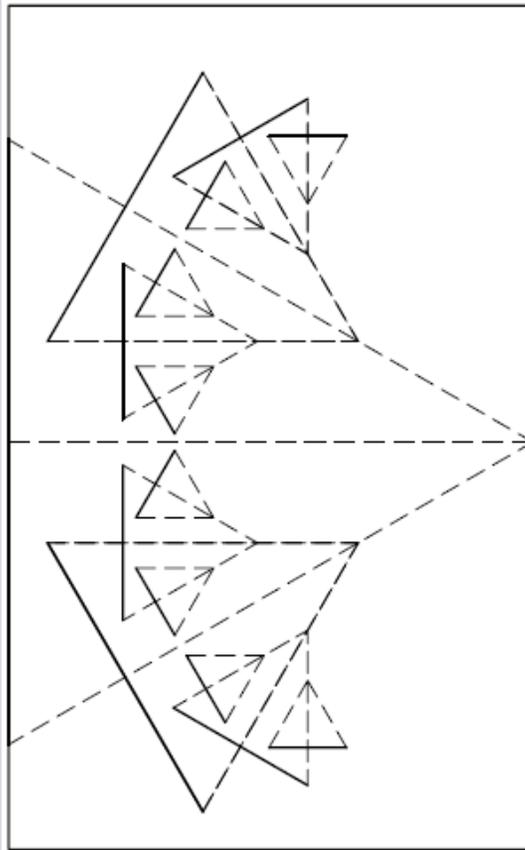


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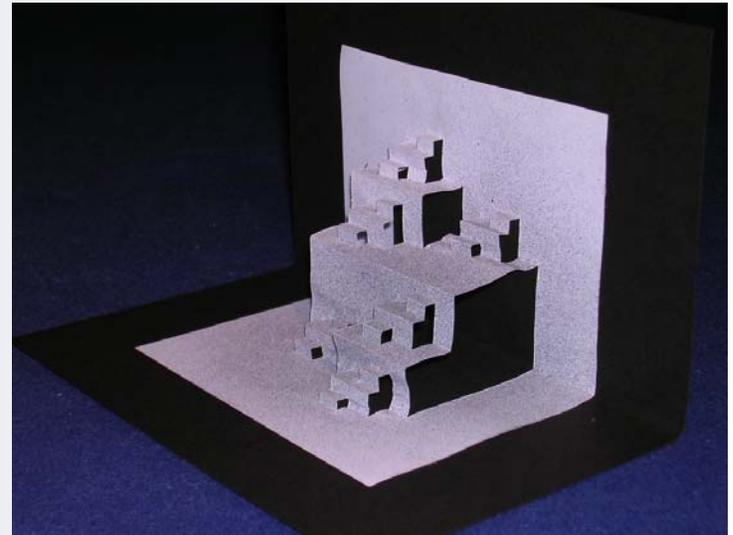
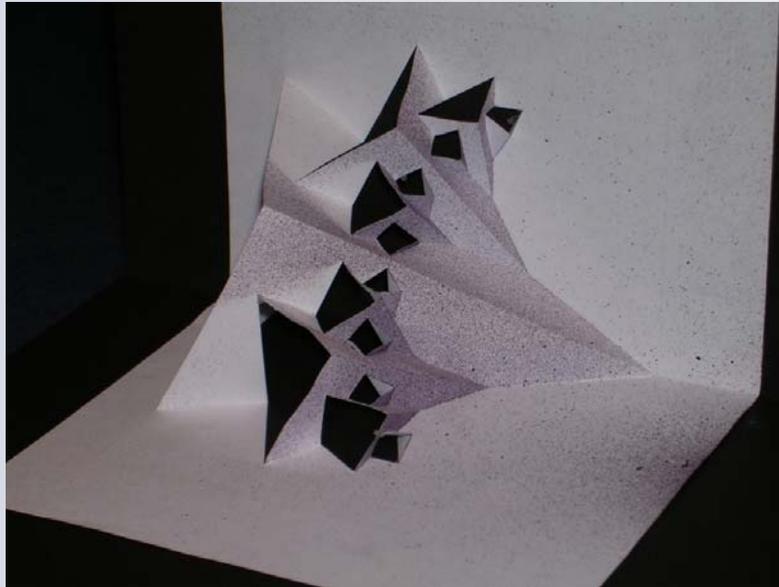
■ Varianten



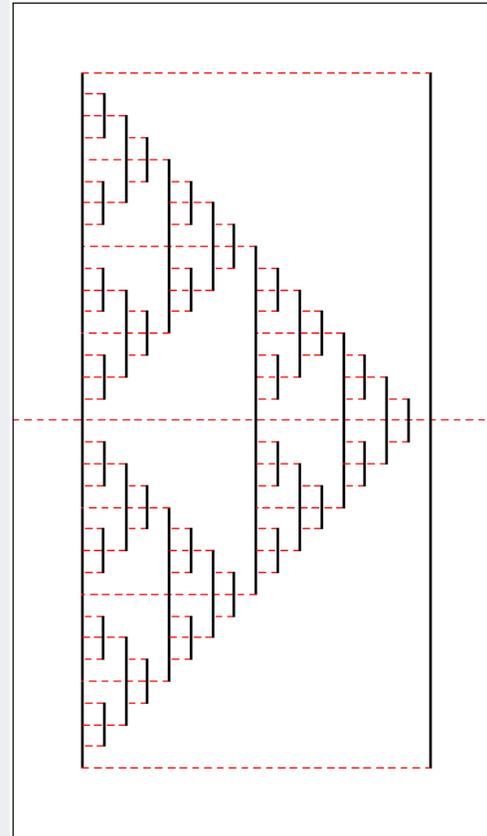
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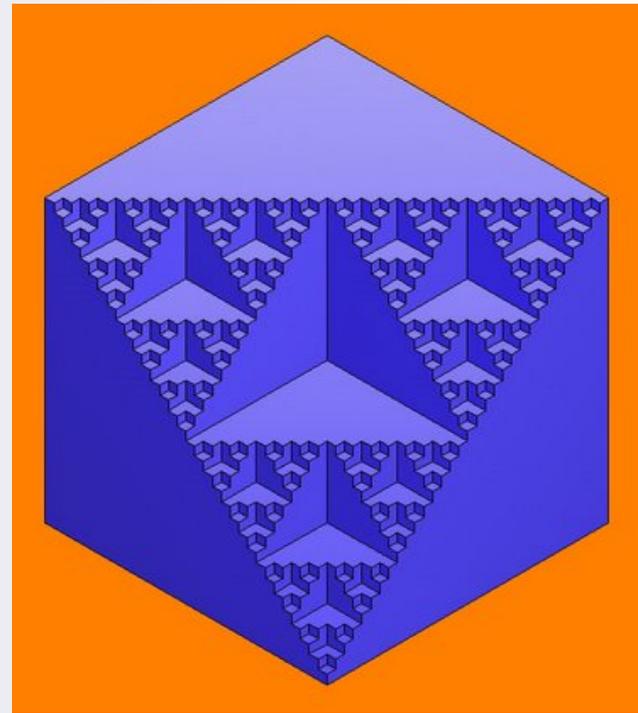
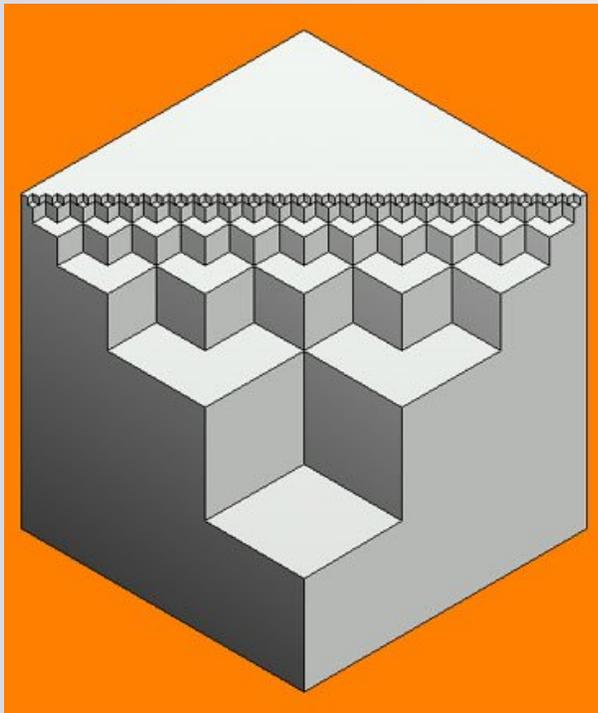
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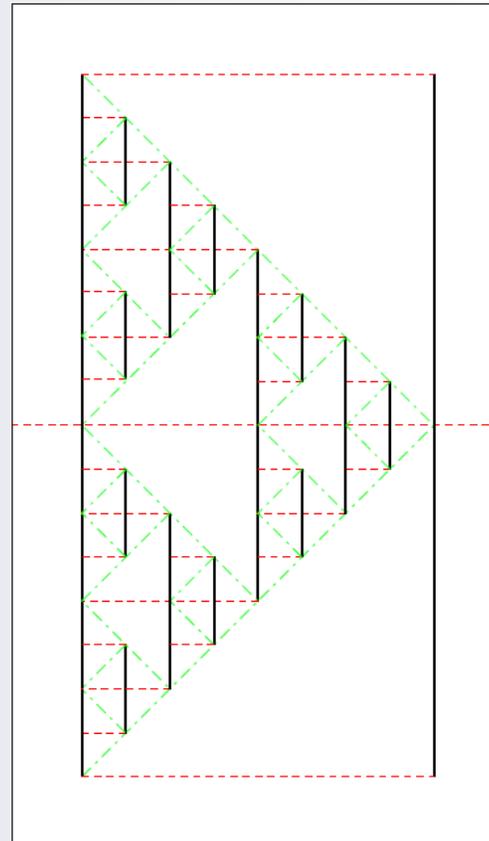
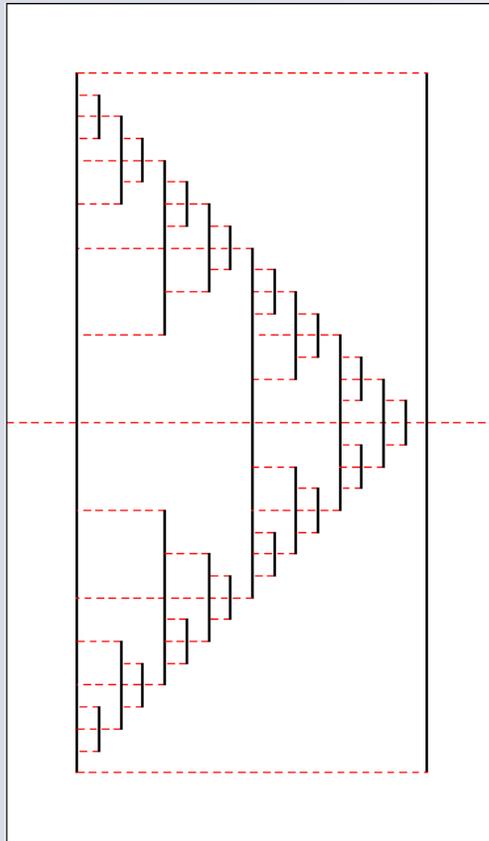
Pop-up-Card



Pop-up-Card



Popup-Card



Pop-up-Card

Nach der Vorzeichnung:

- Linien falzen
 - zB mit einer Zirkelspitze
- Linien schneiden
 - vorzugsweise mit einem Eisenlineal
 - oder entlang der Katheten eines Geo-Dreiecks
- Falten
 - vom Großen ins Kleine
- auf Fotokarton kleben



Literatur

- URIBE Diego: Fractal Cuts. Tarquin Publications, Stradbroke 2004.

